Research Journal of Management Reviews. Vol., 7 (3), 16-29, 2022
Available online at http://www.rjmrjournal.com
ISSN 2149-9168 ©2022
DOI: 10.52547/rjmr.7.3.16

Choosing the best battery for hybrid and electric vehicles using fuzzy AHP and fuzzy TOPSIS theories

Reza Rashmekharim[1*], Ahmad Azizi[2]

[1]Ph.D. Student, Department of Mechanical Engineering, Shahrood University of Technology, shahrood, Iran
[2]Department of Mechanical Engineering, Islamic Azad University, Semnan, Iran

[*]**C**orresponding **A**uthor **E**mail: Rezarashmekharim321@yahoo.com

Abstract: Today, due to increasing number of vehicles, some challenges including emission of greenhouse gas, air pollution caused by vehicle operation, and dependence on fossil fuels endangered the transport sector and the automotive industry. Electric and hybrid vehicles technology is one of the most promising solutions to deal with these challenges. One of important sections in this type of vehicles is portable source of electrical energy storage for supplying their needed electrical energy. The present study aims at selecting the best battery to store and supply electrical energy in electric and hybrid vehicles, such that maximum compatibility and compliance with the criteria set by the manufacturers of these types of vehicles can be achieved. In this study, firstly, a variety of sources for storing and supplying electrical energy in electric and hybrid vehicles were introduced. Secondly, the batteries, their attributes, and their measuring metrics were presented. Thirdly, batteries used in electric and hybrid vehicles and their advantages and limitations were discussed. Finally, since selecting the best battery for electric and hybrid vehicles was basically a problem of multi-criteria decision, we compared and ranked batteries based on criteria provided, using fuzzy AHP and fuzzy TOPSIS theories, and determined the best battery for use in these vehicles.

Keywords: Electric and hybrid vehicle, Choosing the best battery, Fuzzy AHP theory, Fuzzy TOPSIS theory

Introduction

Nowadays, regarding the ever-increasing of the numbers of vehicles and because of the limitations of energy resources and the impact of the internal combustion engines on the air pollution and the greenhouse effect, most research is trending towards using the hybrid renewable energy resources instead of using fossil fuel, and nonrenewable ones in the automotive industries. Consequently, in recent years, the hybrid electric vehicles are recognized as proper replacements for fossil fuel vehicles. In electric vehicles, an electric engine is used instead of an internal combustion one and an electric battery is used instead of a fuel tank.The rechargeable batteries can be charged via plugging in to the electricity , the energy of the vehicle brakes and even non plug in electric resources like fuel and solar cells. The hybrid vehicles are the ones using the hybrid of two or more of power and energy storage resources to supply the necessary power to move. The electric vehicles are categorized into three groups: Hybrid electric vehicles (HEV), Battery electric vehicles (BEV), and Fuel Cell Electric Vehicles (FCEV). The high cost of the batteries of such vehicles, the long time necessary to charge them and low level security and resistance are of problems in the commercialization of electric vehicles. The hybrid vehicles (HV) are new technology in automotive industries which use the hybrid of various energy resources in order to benefit from the advantages of various resources simultaneously. In HVs one of the elements is used as a resource to store energy and the other is used in order to transform it. In HEVs, an internal combustion engine and one or more electric engines are used. Their batteries have the ability of energy absorption from gasoline engines and the engine brakes.The goal of HEVs is achieving the maximum efficiency of fuel, notwithstanding their higher efficiency compared with gasoline vehicles. The Plug in Hybrid Electric Vehicles (PHEV) are another type which possess rechargeable batteries chargeable via external electricity. They have more batteries compared with HEVs. The main difference between batteries of these two types of vehicles is that the ones of PHEVs must be able to be discharged and recharged rapidly while the ones in HEVs work in the condition of almost completely charged and are rarely discharged. Another type of vehicles is Fuel Cell Vehicles (FCV). They are similar to electric ones. The difference is that fuel cells are used as a source to supply energy in them. FCVs are categorized into simple FCs and Fuel Cell Hybrid Vehicles (FCHV)s in the former the cells themselves and the fuel mass are used to supply power and there is no energy supply from batteries. The latter is mainly a type of HEVs which is equipped with fuel cells and use the maximum efficiency of the energy of fuel cells and high power and the ability of fast starting of batteries simultaneously. In the table 1 the advantages and disadvantages of different types of vehicles are indicated.

Table 1. The advantages and disadvantages of various vehicles

Vehicle Types	Advantages and Disadvantages							
	Clean fuel	Low cost	availability	trustworthiness	Long-time function	Dependency on chemical batteries	Constant linking with electric grid	Fast dynamic start and reaction
Internal combustion		×	×	×	×			×
BEV	×			×		×		×
HV	×			×	×			
HEV	×			×	×	×		×
PHEV	×			×	×	×	×	×
FCEV					×	×		
FCHV	×	×	×	×		×		×

Energy storage resources in HEVs

Using apposite resources and methods in order to store energy are of utmost importance in the competition of HEVs with ordinary ones. In this section we introduce and analyze these resources. The most common energy storage resources in HVs are batteries, super capacitors and flywheel energy storages.

Batteries

Batteries are categorized and primary and secondary based on their capacity to be recharged. The primary batteries are not rechargeable and only used to be discharged. The secondary ones are rechargeable and are used in Electric vehicles and HEVs. Batteries have a very salient role in supplying the electric energy in HVs but there are problems using them: considering the up-to date technology, they are very expensive. They have a relatively limited storage capacity and also their life is relatively short. In fact, after a short time (of recharging and discharging), they are not usable anymore and their chemical electrolytes are highly reactive which are a serious danger for the environment. The life of batteries can only be increased via charging them under proper circumstances, and proper degree and in other words via fewer times of charging. Regarding the HEVs. We look for batteries with high specific energy and power and long life and low cost. One of the reasons why PHEVs are not present, is the batteries used in them, since their technology is not yet fully developed, they are expensive and very heavy and have a big volume.

Super capacitators

A super capacitator (also called super condenser, piezo capacitator or two-layer electrochemical capacitator) is a two layer electrical and electrochemical capacitator which has a relatively high specific energy. Compared with an ordinary capacitator, the specific energy in super capacitators is thousands times more than in electrolyte ones. Compared with common batteries and fuel cells, super capacitators have also more specific power. However, their specific energy is very lower than the one of them. The salient characteristic of super capacitators is their ability to be rapidly recharged. These capacitators have a long life cycle and in addition, they have low decrease of power although they can be recharged and discharged many times. As a result, they are somehow considered eco-friendly, since after a short time, batteries are not usable and having highly reactive electrolytes, they are considered hazardous environmental wastes.

Overall the advantages of super capacitators are as follows:

The low cost and high degrees of charges and discharges.

Proper reversibility

High output power and efficiency

Suitable immunity and no corrosive electrolyte and poisonous materials.

The limitations of super capacitators are as follows:

The low specific energy compared with batteries and fuel cells

The process of self-charging

The very high cost and complexity of their management.

Nowadays, super batteries are used to store electric energy and maintain batteries in the heat resistance containers and increasing their lives in many advanced HEVs. Super batteries are unit sets of a super capacitator and a battery which has lower cost and more power in addition to long life. In order to optimize the battery life, we must use them in a slow dynamic cycles. In fast dynamic cycles, we must use capacitators. In fact, in super batteries, super capacitators are main suppliers of energy in first moments of engine accelerations because they have the fastest power reaction and are able to absorb and store energy fast. They are implicitly used as the second energy storage resource in HEVs in order to supply the loading power of batteries. The important issues in maintaining the charge and discharge of super capacitators in super batteries are:

Preventing the flow of super capacitator current when vehicles are receiving energy.

Preventing the flow of battery current to super capacitator when vehicles brake.

Flywheel Energy Storage

Flywheel Energy Storages (FES) are a simple mechanical (kinetic) form of storing energy. The stored energy is done via acceleration or turn of a disc or rotor around itself until it becomes very fast and maintaining energy in the system as a kinetic rounding one. When releasing this energy, the turning speed of FEV decreases and is used in order to accelerate the vehicle. The stored energy is related to the FES rotor inertia moment and the square of its rotating velocity.

$$E= \frac{1}{2} L I^2 \quad (1)$$

In this formula, E is the kinetic energy, L the moment of inertia and l is the rotating speed. In most FES, electricity is used to accelerate and decrease the speed of FES. However, systems using mechanical energy directly are also developing. FES can move mechanically via CVT (Continuously Variable Transmission), cogwheels or engines /generators electrically. The energy of a high Speed FES (maximum 60 thousand turning per minute) has a devastating power. As a result, the cogwheel section must be kept inside a container. In other words, in designing HEVs, FESs must be dynamically secured and the following three points must be taken into account:

-The possibility of breaking of the engine and collision with the container when the vehicle is moving or having an accident

- The Gyroscope effect of FESs can result in vehicle turning upside down when turning.

- The shock resulting from the road can affect the performance of FESs.

- Since if FESs are in automatic mode, they need much power, the necessary voltage is high and usually about 300-500 volts. This high voltage can result in driver, passengers and servicemen being electrically shocked. Therefore, FES need electric immunization. Generally, their advantages and disadvantages are as follows:

Advantages of FES
- Trustworthiness, longer life compared with batteries
- The low cost of repair and maintenance compared with batteries
- Fast charging
- Being eco-friendly

Disadvantages of FES
- Starting capital being higher compared with batteries
- Necessity of dynamic immunization
- Necessity of electrical immunization

Main factors to succeed in development of HEVs are using of energy management system,drawing of energy rings among energy resources (batteries,super capacitators, FESs),determining the duty of each resource and their usage method in various situations. Among the energy storage resources in HEVs, batteries are the first and most common resource for HEVs because they are inexpensive, commercial and lacking mobile parts. In this study, regarding the role of batteries in supply and storage of electric energy in HEVs, different types of batteries used in them and their features and measurements are presented. The best battery for them is also selected.

The features and measurement units of batteries

In order to compare batteries and choose the best one for HEVs, it is necessary to introduce their features:

Specific Energy: The energy (stored in batteries) which can be received per the unit of mass which is indicated by the unit of Watt per hour per Kilogram (Wh/Kg)

Specific Power: The power amount (stored in batteries) which can be received per kilogram in a definite depth which is indicated by the unit of Watt per Kilogram (W/Kg)

Life cycle: Charging and discharging batteries, their overall receivable capacity decrease. Life cycle shows the number of charging and discharging cycles per Depth of Discharge (DOD). In other words, it indicates the maximum number of (efficient) charge and discharge cycles of batteries before reaching the DOD of replacement.

The Depth of Discharge (DOD): The amount of discharging. In fact, it is indicated as a percentage of battery capacities. If it is 0 percent it means that DOD is zero and if it is eighty percent, it means that eighty percent of the primary battery energy is consumed.

c-rate: a scale for measuring and comparing the charge velocity of batteries. Its formula is as follows

c-rate = (Current of charge or discharge per milli ampere/Rated energy (per milli ampere)

For example, 1C means the whole battery will be completely discharged in one hour.

Self-Discharge per month (%/month): If a battery is not used at all, rated power equal to a little thousandth is wasted automatically. Some factors affecting it are battery materials, technology and high temperature of environment and exhaustion.

Efficiency: The Ampere hour efficiency of batteries is the consumed ampere hour in order to fill batteries in relation to the delivered one regarding their load. The power efficiency is the relationship between the injected power and the delivered one in batteries. This is shown by percentage.

Usage: According to the length of time and distance in which vehicles are used batteries are categorized as short-term midterm and long term.

Cost: This is based on dollars and indicates the amount of their cost for production and consumption. This is indicated by unit of dollars per kilo watt per hour (cost/kwh)

Availability: This shows how available batteries are in markets and how common their production and consumption is in HEVs.

Other features of batteries are the temperature of performance and their limitation in them , immunity, reversibility, eco-friendliness, life and decrease in charge time.

In fact, we always need batteries which have high specific energy and storage capacity, long and stable half-life (long life cycle), high power and efficiency and can be rapidly and inexpensively charged.

Batteries used in HEVs

In this section, some batteries which are common in HEVs are introduced:

Lead-acid batteries: These batteries are most common in HEVs. Their positive electrode is made of lead oxide and the negative one is made of lead metal. The electrolyte used in contains sulfuric acid with the concentration of about 25 to 40 percent and water with the concentration of about 60 to 75 percent. The reactions in the cells of these batteries in the process of charging and discharging are shown in table (2). They are easy to produce, inexpensive highly available. Their self-discharge is also low. The disadvantages are that their energy storage is low and their complete discharge times are limited. They are also not eco-friendly. When charging is being used, their temperature increases very much. In recent years, these batteries are often used in short-term applications, e.g. when using wheeled chairs.

Table 2. The reactions occurring when charging and discharging in lead-acid battery cells

Charge		Discharge	
(-) Cathode	(+) Anode	(-) Cathode	(+) Anode
$PbSO_4+H^++2e^-\longrightarrow Pb + HSO_4^-$	$PbSO_4+2H_2O \longrightarrow PbO_2+ HSO_4^- +3H^+ + 2e^-$	$Pb+HSO_4^-\longrightarrow PbSO_4+H^++2e^-$	$PbO_2+HSO_4^- +3H^+ +2e^-\longrightarrow PbSO_4+2H_2O$
The overall Battery reaction		The overall Battery reaction	

2 $PbSO_4$+2 H_2O ⟶Pb+ PbO_2+2 H^+ + 2 HSO_4^-	Pb+ PbO_2+2 H^++2 HSO_4^-⟶ 2 $PbSO_4$+2 H_2O

Nickel Cadmium Batteries: These batteries are also one of the other common ones used in HEVs which have mostly short-term applications. In their cells the positive electode is made of nickel hydride and the negative one is made of sponge Cadmium. The dilution of Potassium Hydroxide in water is used as electrolyte. The reaction in their cells is as follows:

$$CD + 2NiOOH + 2H_2O \underset{charge}{\overset{discharge}{\rightleftarrows}} 2Ni(OH)_2 + CD(OH)_2 \quad (2)$$

Their advantages are being inexpensive and highly available. Also their maintenance and transportation are simple and their charge velocity is higher compared with lead-acid batteries. Their disadvantages are that their application is very expensive and that they are not eco-friendly.

Nickel-metal Hydride Batteries: These batteries are used in order to compensate for disadvantages of the Nickel-Cadmium ones. They are to supply more energy and be recharged and better reused. In their cells , the Cathode includes the hydroxide of Nickel and the Anode is made of Hydride which determines the life of cells. The separator made of fibers with electrodes which conducts ions. Their electrolyte used in cells is in the form of concentrated dilution of Potassium Hydroxide (KOH). This electolyde can conduct various temperatures very well. Four reactions occur in their cell two of which are in the section of positive electrode (Cathode) and the other two are in the one of the negative electrode (Anode).

$$NiOOH + H_2O + e^- \underset{charge}{\overset{discharge}{\rightleftarrows}} Ni(OH)_2 + OH^- \quad (3)$$

$$2OH^- \rightarrow \frac{1}{2}O_2 + H_2O + 2e^- \quad (4)$$

$$MH + OH^- \underset{charge}{\overset{discharge}{\rightleftarrows}} H_2O + M + e^- \quad (5)$$

$$\frac{1}{2}O_2 + H_2o + 2e^- \rightarrow 2oH^- \quad (6)$$

In each electrode a primary and a secondary reaction occur. The primary reaction in Cathode, is the oxidation of active materials in Nickel. In Anode is the decrease of the metal materials of the Hydride. The secondary reaction in Cathode is the oxidation of oxygen and in Anode is its decrease. Two secondary reactions are coupled via transferring oxygen from Cathode to Anode. Compared with Nickel-Cadmium batteries, these batteries are eco-friendlier and about 30 to 40 percent better than them. They are mostly applied mid-termly. Their disadvantages are that they have shorter lives, lower discharge currents they also need a complicated charge algorithm since they emit heat when used or charged.

Lithium Ion Batteries: Lithium is a metal with a very high specific energy. In recent years. using Lithium batteries have become very common. The Lithium Ion batteries are the most common types of Lithium ones. Their most common structure is formed by a graphite Anode, a cathode possessing an Oxide of Lithium ($limo_2$ like $lico_2$) and an electrolyte which is a dilution of a Lithium salt (e,g $LiPF_6$) in an organic combination (e.g. Ethylene carbonate-Dimethyle Carbonate)(EC-DMC). A plate separates them. The electrode to be used in a Lithium battery must be highly reactive and electrically conductive. It must also have fast electrochemical reactions and stable structure. The most common Anode material used in Lithium Ion batteries is carbon. The reason is that carbon is highly recyclable and is very stable and has a long life. Graphite is a carbon structure which is very electrically conductive inside layers and is used as Anode in Lithium batteries. Instead their electrical conduct between layers is very low. This feature results in Lithium being easily placed between their layers and perform the electron exchange reaction. The Cathodes used in these batteries are ususally categorized into three groups: the basic metal oxides of Lithium e.g. $Lico_2$, the phosphates of transition metals (e.g $LiFepo_4$, $Li_3V_2(PO_4)_3$) and oxide one metal composites with Lithium. The performance of a Cathode material is controlled via its structural stability and conductivity. In addition, just like Anodes, Cathodes should also be able to receive and release Lithium in a reversible way. Therefore, the penetration of Io of Lithium in the Cathode is of utmost importance. Electrolytes used in Lithium Ion batteries should be separable in solvent system so that Lithium Ions can penetrate in these systems very well. The common electrolytes usually Lithium salts diluted in organic solvents. One of the apposite materials for this purpose is carbonate propylene. They have a high dielectric constant, high temperature ranges in liquid and compatibility with Lithium. Therefore, they have come under much consideration recently. If we suppose that a sample of Lithium Ion battery possess the positive electrode of Oxide Cobalt Lithium and Graphite negative electrode. In the process of charging electrode half reaction is:

$$LiCo_2 \rightarrow Li_{1-x}CoO_2 + xLi^+ + Xe \quad (7)$$

In the negative electrode the half reaction is as follow:

$$x\,Li^+ + xe + c_6 \rightarrow LiC_6 \quad (8)$$

While charging, the Lithium in the structure of the positive electrode is changed into ions and the ions in the electrolyte are stored between carbon(graphite) layers. This process necessitates electricity. when discharging, the reverse of the abovementioned reaction occurs and the stored electricity is released. The advantages of these batteries are high specific energy, very high efficiency and very low self-discharge. The disadvantages are that they are expensive and their circuits to protect them are complex.

Lithium Polymer Batteries: These are a type of Lithium batteries made when their technologies advanced. In recent years, these batteries have become very common. Their advantages are that they are smaller and lighter and have a longer life cycles compared with Lithium ones. Their disadvantages are that they are expensive and hardly available.

Lithium Iron Phosphate Batteries: These batteries are Lithium ones in which Cathode is Lithium Iron Phosphate. Their advantages are that they are lighter and have a longer life cycles compared with Lithium ones. Their disadvantages are that they are expensive and hardly available. Table (3) indicates various types of batteries used in HEVs:

Table 3. The comparison among batteries used in HEVs [8,20,21]

Battery	Nominal voltage	Specific energy (wh/kg)	Specific Power (Kw/Kg)	The self-discharge in month (%)	Efficiency(%)	Life cycle	usage	Cost(dollars)	availability
Lead-acid	2.1	30-40	0.1-0.18	5	70-80	Upto 800	short-term	20-120	Very high
Nickel-Cadmium	1.2	40-60	0.3-1.2	20	70-80	Upto 1000	short-term	200-300	High
Nickel-metal Hydride	1.2	30-80	0.4-1.2	30	60-70	Upto1000	mid-Term	250-300	High
Lithium Ion	3.6	150-250	0.8-2	<5	95	Upto1200	long-term	200-1000	Average
Lithium Polymer	3.7	130-200	1-2.8	<5	80-95	Upto1000	long-term	250-1000	Average
Lithium Iron Phosphate	3.25	80-200	1.3-3.5	<5	93	Upto2000	long-term	350-1000	Low

The aim of this study is selecting the best type of battery for HEVs regarding their importance in such types of vehicles. Selection of proper batteries for HEVs is mainly a multiple-scale decision. This issue is of strategic importance for battery producers and automotive industry men. In this study, using the fuzzy AHP and TOPSIS theories and regarding the determined scales and features of batteries in HEVs, the batteries are compared and the best one is selected.

Evaluating measures, features and choices based on multiple scale decision making

The models of multiple –scale decision making are categorized as multiple goal ones and multiple scale ones [sic.]. In the former category, multiple goals are considered in order to simultaneously optimize outcomes. The measuring scales for each goal could be different with each other. For example, a goal could be maximizing profits which is measured by money unit. The other one could be the minimum amount of use of work hours. On the other hand, sometimes these goals could oppose each other and not progress in the same way. In this regard, the most beneficent method for making decisions is ideal planning. Because of the importance of multiple scale decision making models and their use, some explanations will be given in the subsequent sections. This kind of decision making is a group of techniques and methods in making decision which is used in order to prioritizing and/or selecting the most proper choice among m choices based on n scales. Such problems are usually formulized like table 4.

Table 4. The Overall Decision Matrix

X1	X2	Xn	معیار / گزینه
r11 r1n r21 r2n ⋮ rm1 rmn	r12 .... r22 .... ⋮ rm2	⋮	A1 A2 ⋮ Am

In this matrix, a indicates the jth choice and rij indicates the evaluation of ith choice based on jth scale. The better choice is the one fulfilling the ideal of all scales or features. But it is impossible most of the time. Nevertheless, mathematically the best choice in a multiple decision making model is a hypothetical A^* which is considered the most highly valued or preferred per each scale. The problems in such decision making models are various. But they have the following common features:

Choices: In these decision making models some limited choices are observed for prioritizing, choosing or categorizing. Choice is usually synonym to selection, attitude, method, performance or candidate.

Multiple scales and features: Each problem has multiple scales and features. These are presented by decision maker(s) and choices are evaluated according to them. At the end, the best choice is selected and prioritized. The number of features depends on the nature of the problem. For instance, a person may use features like price, amount of consumed fuel, immunity and the period of guarantee and the quality of manufacturing when evaluation of purchasing a car. Another person may consider more than one hundred scales and features for choosing the location of a factory.

Incommensurable units: Each scale is measured differently compared with others. Therefore, since calculations and result are scientifically meaningful, data have been made incommensurable in a way to preserve their relative importance.

The feature weight: All such decision making methods necessitate information achieved based on the relative importance of each feature. This information usually has ordinal or main scales. Weights attributed to each feature can be identified directly by the decision makers or via scientific methods. In fact they indicate the extent of relative importance of each feature in making decisions.

The fuzzy AHP theory

Since deciders mostly fail in making a good numerical anticipation for features, measurement is stated linguistically. In addition, human judgment has mental and vague trend in qualitative features. As a result, the fuzzy AHP theory is generally used in making decisions. The fuzzy numbers are developed regarding the idea of trust break and defined in the whole of a fuzzy subcategory of real numbers.

The fuzzy number $\tilde{M}$ is defined in relation to R is a TFN, if the membership function $\mu_{\tilde{M}}(y)R \rightarrow [0,1]$ is equal to:

$$\mu_{\tilde{M}}(y) = \begin{cases} (y-1)/(m-1) & l \leq y \leq m \\ \frac{u-y}{u-m} & m \leq y \leq u \\ 0, & otherwise \end{cases} \quad (9)$$

Otherwise L,u,m are real numbers and $l \leq y \leq m$. The linguistic scale of the variables and the relative TFNs used in this study are shown in Table 5.

In order to achieve the feature weights using fuzzy AHP, first the paired comparison of features is formed and experts give their opinions regarding it. After harmonizing all decision makers' opinions regarding the paired comparison according to the goal of this problem via the geometrical mean, the following stages are followed:

Table 5. The vocabulary for expressing the importance and preference of variables

Vocabulary for preference and importance	TFN
Equal Preference	(1,1,1)
Average	(1,2,3)
A little preferred	(2,3,4)
Average	(3,4,5)
Very preferred	(4,5,6)
Average	(5,6,7)
Extremely preferred	(6,7,8)
Average	(7,8,9)
Completely Preferred	(9,9,9)

$$Si = \sum_{j=1}^{m} M_{gi}^{j} \left[\sum_{i}^{n} \sum_{j=1}^{m} M_{gi}^{j} \right]^{-1} \quad (10)$$

$$\sum_{j=1}^{m} \sum_{gi}^{j} = \left(\sum_{j=1}^{m} l_{ij}, \sum_{j=1}^{m} m_{ij}, \sum_{j=1}^{m} u_{ij} \right) \quad ,i = 1,2, \dots \quad (11)$$

$$\sum_{j=1}^{n} \sum_{j=1}^{m} M_{gi}^{i} = \left(\sum_{i=1}^{n} \sum_{i=1}^{m} l_{ij}, \sum_{i=1}^{n} \sum_{i=1}^{m} m_{ij}, \sum_{i=1}^{n} \sum_{i=1}^{m} u_{ij} \right) \quad (12)$$

$$\left[\sum_{i=1}^{n} \sum_{i=1}^{m} M_{gi}^{j} \right]^{-1} = \left(\frac{1}{\sum_{i=1}^{n} \sum_{i=1}^{m} l_{ij}}, \frac{1}{\sum_{i=1}^{n} \sum_{i=1}^{m} m_{ij}}, \frac{1}{\sum_{i=1}^{n} \sum_{i=1}^{m} u_{ij}} \right) \quad (13)$$

In order to calculate how M_2 is bigger compared with M_1 , the following formulas are used [24]:

$$V(M_2 \geq M_1) = \sup_{y \geq x}[\min(\mu_{M_1}(x), \mu_{M_2}(y)] \quad (14)$$

$$V(M_2 \geq M_1) = hgt(M_2 \cap M_1) = \mu_{M_1}(d) = \begin{cases} 1, if\ m_2 \geq m_1 \\ 0, \quad if\ l_1 \geq u_2 \\ \dfrac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)} \end{cases} \quad (15)$$

$$(M \geq M_1, M_2, \dots\dots, M_k) = V[(M \geq M_1) and (M \geq M_2) and \dots and (M \geq M_{k)}] = \min V\ (M \geq M_i), i = 1,2, \dots k \quad (16)$$

$$d'(A_i) = \min V\ (s_i \geq s_k) K = 1,2, \dots, n; k \neq i. \quad (17)$$

The weight vector is calculated via using the following formula:

$$W' = \left(d'(A_1)\right), d'(A_2), \dots, d'(A_n))^T A_i (i = 1,2, \dots, n) \quad (18)$$

Finally, using the following formula,the normalized weight is calculated:

$$W = \frac{w_i'}{\sum w_i'} \quad (19)$$

First, the matrix of paired comparison the features of batteries is formed. Then, the final weight of each feature is calculated based on the fuzzy AHP theory, In the table 6 the desired features are marked and the table 7 , the paired comparison of battery feature matrix is indicated using the experts' opinions.

Table 6. The Feature of Batteries in Symbols

Feature	Specific Energy	Specific Power	Cycle	Efficiency	Usage	The self-discharge in month	availability	Cost
Symbol	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8

Table 7. The Paired comparison matrix

feature / feature	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
C_1	(1,1,1)	(1,2,3)	(2,3,4)	(2,3,4)	(2,3,4)	(2,3,4)	(2,3,4)	(1,2,3)
C_2	(1,0.5,0.33)	(1,1,1)	(1,2,3)	(1,2,3)	(1,2,3)	(2,3,4)	(1,2,3)	(1,2,3)
C_3	(0.5,0.33,0.25)	(1,0.5,0.33)	(1,1,1)	(1,2,3)	(1,2,3)	(1,2,3)	(1,0.5,0.33)	(1,0.5,0.33)
C_4	(0.5,0.33,0.25)	(1,0.5,0.33)	(1,0.5,0.33)	(1,1,1)	(1,2,3)	(1,2,3)	(1,2,3)	(1,2,3)
C_5	(0.5,0.33,0.25)	(1,0.5,0.33)	(1,0.5,0.33)	(1,0.5,0.33)	(1,1,1)	(1,1,1)	(1,0.5,0.33)	(1,0.5,0.33)
C_6	(0.5,0.33,0.25)	(0.5,0.33,0.25)	(1,0.5,0.33)	(1,0.5,0.33)	(1,0.5,0.33)	(1,0.5,0.33)	(1,0.5,0.33)	(1,0.5,0.33)
C_7	(0.5,0.33,0.25)	(1,0.5,0.33)	(1,2,3)	(1,0.5,0.33)	(1,2,3)	(1,2,3)	(1,1,1)	(1,0.5,0.33)
C_8	(1,0.5,0.33)	(1,0.5,0.33)	(1,2,3)	(1,0.5,0.33)	(1,2,3)	(1,2,3)	(1,2,3)	(1,1,1)

Now, according to the paired comparison of features matrix and the fuzzy AHP theory, the final weight of choices is indicated in table 8.

Table 8. The Final Weight of the Features of Batteries

C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
0.214804	0.176767	0.116747	0.13564	0.069377	0.03101	0.116747	0.1389

The Fuzzy TOPSIS Theory

TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) is recognized as one of the traditional methods for multiple decision making. Based on determination of the ideal, this method was developed in 1981 by Hwang and Yoon in order to solve problems of multiple feature decision makings. In this technique, the selected choice must be the closest to the positive ideal and the farthest from the negative one. The usage of this model in Iran began since 1991 in a limited way. Its use is limited to recent years.

The stages in decision making using the fuzzy TOPSIS theory are as follows:

Stage 1- Calculating the vectors of weights *W~j*

Stage 2-Normalization of the calculated matrix regarding the choices using the experts' opinion: The new matrix is as follows:

$$\tilde{R} = [\tilde{r}_{ij}]_{m\times n} \qquad i = 1,2,\dots,m \qquad j = 1,2,\dots.n \tag{20}$$

$$\tilde{r}_{ij} = \left(\frac{a_{ij}}{c_j^*}, \frac{b_{ij}}{c_j^*}, \frac{c_{ij}}{c_j^*}\right) \tag{21}$$

Stage 3- Therefore the weighted matrix is indicated via the following formula:

$$\tilde{V}_{ij} = \tilde{r}_{ij} \otimes \tilde{w}_j \tag{22}$$

$$\tilde{V} = [\tilde{v}_{ij}]_{m\times n} \qquad i = 1,2,\dots m \qquad j = 1,2,\dots,n \tag{23}$$

Stage 4- Determination of the ideal positive and negative fuzzy solutions:

$$A^+ = (v_1^*, v_2^*, \dots, v_n^*) \tag{24}$$

$$A^- = (v_1^-, v_2^-, \dots, v_n^-) \tag{25}$$

Stage 5- Calculation of differences in numbers using the fuzzy Euclidean distance:

$$\tilde{A} = (a_1, a_2, a_3) \tag{26}$$

$$\tilde{B} = (b_1, b_2, b_3) \tag{27}$$

$$D(\tilde{A}, \tilde{B}) = \sqrt{\frac{1}{3}[(a_2 - a_1)^2 + (b_2 - b_1)^2 + (c_2 - c_1)^2]} \tag{28}$$

Stage 6: The distances between each positive and negative ideal are calculated via these formulas:

$$d_i^* = \sum_{j=1}^{n} d\left(\tilde{v}_{ij} - \tilde{v}_{ij}^*\right) \qquad i = 1,2,\dots,m \tag{29}$$

$$d_i^- = \sum_{j=1}^{n} d\left(\tilde{v}_{ij} - \tilde{v}_{ij}^-\right) \qquad i = 1,2,\dots,m \tag{30}$$

Stage 7: The calculation of the relative closeness to the ideal and ranking:

$$CC_i = \frac{d_i^-}{d_i^* + d_i^-} \qquad i = 1,2,\dots,m \tag{31}$$

Now, for each feature,the paired comparisons are formed using the experts' opinions via the fuzzy TOPSIS matrix .the weights of each battery is calculated per each feature and the overall decision matrix is achieved. The table (9) shows the batteries and their symbols.

Table 9. The Batteries used in HEVs and their Symbols

Battery	Lead-Acid	Nickel-Cadmium	Nickel-metal Hydrid	Lithium Ion	Lithium Polymer	Lithium Iron Phosphate
Symbol	A_1	A_2	A_3	A_4	A_5	A_6

Using the Fuzzy TOPSIS theory, the results of comparing batteries for each feature is indicated in the tables 10-17.

Table 10. The final Weight of batteries per the feature of Specific Energy

Specific Energy	d_i^*	d_i^-	CC_i	The final Weight
A_1	5.037081	0.357278	0.066232	0.030239
A_2	4.632107	0.830602	0.152049	0.069421
A_3	4.321693	1.19556	0.216695	0.098936
A_4	1.764359	4.120397	0.700181	0.319682
A_5	2.48336	3.427186	0.579843	0.264739
A_6	3.029166	2.743364	0.475245	0.216972

Table 11. The final Weight of batteries per the feature of Specific Power

Specific Power	d_i^*	d_i^-	CC_i	The final Weight
A_1	5.180082	0.284294	0.052027	0.025856
A_2	4.86356	0.646463	0.117325	0.058307
A_3	4.605545	0.93843	0.16927	0.084122
A_4	3.333905	2.42015	0.420599	0.209024
A_5	2.803804	3.069822	0.522645	0.259737
A_6	1.592226	4.312329	0.730339	0.362955

Table 12. The final Weight of batteries per the feature of Life Cycle

Life Cycle	d_i^*	d_i^-	CC_i	The final Weight
A_1	5.003737	0.417614	0.077031	0.040601

A_2	4.571689	0.930894	0.169174	0.089167
A_3	4.261274	1.295852	0.233187	0.122907
A_4	3.450669	2.204698	0.389842	0.205475
A_5	3.950859	1.660811	0.295957	0.155991
A_6	1.549193	4.2330029	0.732076	0.385858

Table 13. The final Weight of batteries per the feature of Efficiency

Efficiency	d_i^*	d_i^-	CC_i	The final Weight
A_1	4.321693	1.208699	0.218556	0.098255
A_2	4.632107	0.848374	0.154084	0.06271
A_3	5.069903	0.333567	0.061732	0.027752
A_4	1.764359	4.135538	0.700951	0.315122
A_5	2.48336	3.442328	0.580916	0.261159
A_6	2.855315	2.949807	0.508139	0.228441

Table 14. The final Weight of batteries per the feature of Usage

Usage	d_i^*	d_i^-	CC_i	The final Weight
A_1	4.726666	0.714256	0.131275	0.060133
A_2	4.890122	0.48472	0.090183	0.04131
A_3	4.321693	1.167164	0.212642	0.097405
A_4	1.764359	4.092566	0.698757	0.32008
A_5	2.48336	3.399355	0.577855	0.264698
A_6	3.029166	2.711774	0.472357	0.216373

Table 15. The final Weight of batteries per the feature of Self Discharge Percentage Per Month

The Self Discharge Percentage per month	d_i^*	d_i^-	CC_i	The final Weight
A_1	3.334853	2.418115	0.420325	0.177656
A_2	4.662317	0.807624	0.147648	0.062405
A_3	5.132934	0.260602	0.048317	0.020422
A_4	1.871942	4.096368	0.686353	0.290097
A_5	2.590943	3.403157	0.567751	0.239968
A_6	2.962898	2.910636	0.495551	0.209452

Table 16. The final Weight of batteries per the feature of Availability

Availability	d_i^*	d_i^-	CC_i	The final Weight
A_1	1.377061	4.492241	0.765379	0.393052
A_2	3.162636	2.564694	0.447799	0.229962
A_3	3.473051	2.195304	0.387291	0.198889
A_4	4.502694	1.085484	0.194247	0.099753
A_5	4.922935	0.644579	0.115775	0.059455
A_6	5.307014	0.202653	0.036781	0.018889

Table 17. The final Weight of batteries per the feature of Cost

Cost	d_i^*	d_i^-	CC_i	The final Weight
A_1	1.484644	4.421929	0.748646	0.374236
A_2	2.913535	2.938388	0.502123	0.251003
A_3	3.42486	2.236884	0.395087	0.197498
A_4	4.473006	1.101398	0.197581	0.098768
A_5	4.893247	0.660493	0.118928	0.05945
A_6	5.285426	0.209355	0.038101	0.019046

At the end, after achieving the battery weights per each feature,the multiple feature decision making matrix is written regarding the abovementioned issues. This matrix is shown in the following table:

Table 18. The final multiple feature decision making matrix

The Feature Weight	0.214804	0.176767	0.116747	0.13564	0.069377	0.03101	0.116747	0.1389
Feature Choice	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
A_1	0.030239	0.0255856	0.040601	0.098255	0.60133	0.1776566	0.393052	0.374236
A_2	0.069421	0.58307	0.089167	0.069271	0.04131	0.062405	0.229622	0.251003
A_3	0.098936	0.084122	0.122907	0.027752	0.097405	0.020422	0.198889	0.197498
A_4	0.319682	0.209024	0.205475	0.315122	0.32008	0.290097	0.099753	0.098768
A_5	0.264739	0.259737	0.155991	0.261159	0.264698	0.239968	0.059455	0.05945
A_6	0.216982	0.362955	0.385858	0.228441	0.216373	0.209452	0.018889	0.019046

After wards, based on the final multiple feature decision making matrix, we have achieved the final weight and rankings of batteries. In the table 19 the final weight and rankings of batteries are indicated:

Table 19. The final weight and rankings of batteries

Battery	The final normalized Weight	The final ranking
Lead-Acid	0.128193345	Fifth
Nickel-Cadmium	0.1689466320	Fourth
Nickel-metal Hydrid	0.1053041952	Sixth
Lithium Ion	0.217079845	First
Lithium Polymer	0.1920246025	Second
Lithium Iron Phosphate	0.1883123178	Third

As indicated above, Lithium batteries have an opposite performance in HEVs. Also, we saw that the Lithium Ion batteries are chosen as the best ones for such vehicles regarding the presented features.

Conclusion

In this study, first different types of HEVs were introduced based on their electric supply resources. Afterwards, batteries, super capacitators, Flywheel Electric Storages (FES) and their advantages and disadvantages in storing and supplying electricity in HEVs were introduced. Subsequently, different types of common batteries in HEVs and their features were presented regarding the importance of battery roles in storing and supplying electricity in such vehicles. Finally, regarding the features used by batteries and HEV suppliers, we compared and ranked batteries via using theories of fuzzy AHP and fuzzy TOPSIS and the experts' comments. As a result, Lithium batteries were chosen as the best ones to use in HEVs.

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